Illustration of Deep Water Linearity
Assumptions in the Non-linear Shallow Water Equations

- Long wavelength compared to the bottom depth.

- Uniform vertical profile of the horizontal velocity components.

- Hydrostatic pressure conditions.

- Negligible fluid viscosity.
Characteristic Form of the 1D Non-linear Shallow Water Equations

\[
\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = 0
\]
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} = g \frac{\partial d}{\partial x}
\]
\[
\frac{\partial p}{\partial t} + \lambda_1 \frac{\partial p}{\partial x} = g \frac{\partial d}{\partial x}
\]
\[
\frac{\partial q}{\partial t} + \lambda_2 \frac{\partial q}{\partial x} = g \frac{\partial d}{\partial x}
\]

Riemann Invariants:
\[
p = u + 2\sqrt{gh}
\]
\[
q = u - 2\sqrt{gh}
\]

Eigenvalues:
\[
\lambda_1 = u + \sqrt{gh}
\]
\[
\lambda_2 = u - \sqrt{gh}
\]

Any term in the equations containing products of the unknowns or their derivatives will cause non-linear behavior.
Characteristic Form of the 1D Non-linear Shallow Water Equations

\[
\frac{\partial p}{\partial t} + \lambda_1 \frac{\partial p}{\partial x} = g \frac{\partial d}{\partial x}
\]

\[
\frac{\partial q}{\partial t} + \lambda_2 \frac{\partial q}{\partial x} = g \frac{\partial d}{\partial x}
\]

Eigenvalues:

\[
\lambda_1 = u + \sqrt{gh}
\]

\[
\lambda_2 = u - \sqrt{gh}
\]

Typical Deep Water Values:

\[
u \approx 0.2 m/sec
\]

\[
\sqrt{gd} \approx 220 m/sec
\]

\[
\lambda_1 = \lambda_2 \approx \sqrt{gd}
\]
Illustration of Deep Water Linearity
Illustration of Deep Water Linearity
Linearity allows for the reconstruction of an arbitrary tsunami source using elementary building blocks.
Unit source deformation

Forecasting Method
Forecasting Method

Locations of the unit sources for pre-computed tsunami events.

West Pacific

East Pacific
Forecasting Method

Unit source propagation of a tsunami event in the Caribbean

Some Future Date: 11:00
Tsunami Warning: DART Systems
Forecasting Method: DART Positions
Forecasting Method: Inversion from DART
Forecasted Max Amplitude Distribution (Japan 2010)
Community Specific Forecast Models
Inundation Forecast Model Development
Forecasting Challenges: Definition of Tsunami Initial Conditions

Tsunami inversion based on satellite altimetry: Japan 2010
Forecasting Challenges: Definition of Tsunami Initial Conditions